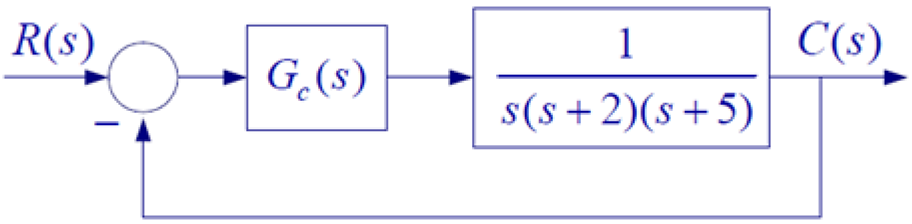
AMERICAN UNIVERSITY OF BEIRUT
ELECRICAL & COMPUTER ENGINEERING DEPARTMENT

EECE 460	**Control Systems**	**Fall 2015**
	Quiz II-Solution	**November 13, 2015**

Problem 1 (12 pts)

Consider the control system shown in the Figure below



a. You are required to design a phase lead componsator to meet the following time-domain specification
   - Dominant poles damping ration is 0.707
   - Dominant Poles rise time constant is 1.176 seconds

   Determine the location of the dominant poles. (4 pts)

Solution

The quadratic approximation for normalized rise time for a 2nd-order system, step response, no zeros is:

$$t_r \cdot \omega_0 = 2.230\zeta^2 - 0.078\zeta + 1.12$$

where ζ is the damping ratio and ω_0 is the natural frequency of the network.

However, the proper calculation for rise time from 0 to 100% of an under-damped 2nd-order system is:

$$t_r \cdot \omega_0 = \frac{1}{\sqrt{1-\zeta^2}}\left(\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$$

where ζ is the damping ratio and ω_0 is the natural frequency of the network.

$$\varsigma = 0.707$$

$$T_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}{\omega_n\sqrt{1-\varsigma^2}} = 1.176 \Rightarrow \omega_n = 2.832871\, rad/s$$

Dominant poles are located at

$$s_{1,2} = -\varsigma\omega_n \pm j\omega_n\sqrt{1-\varsigma^2} = 2 \pm j2$$

b. If the dominant poles are located at *s=-2+j2* and *s=-2-j2*, dtermine the angle deficiency Φ. **Show your steps very clearly**.

$$\Phi - (angle \text{ of the zero pole - the angle of the s = -2 pole - angle of the -5 pole}$$

$$\Phi - 135 - 90 - 33.69 = -180$$

$$\Rightarrow \Phi = 78.69^0$$

c. Determine the componsator transfer function by determing the exact locations of the pole and the zero. (4 pts)

$$G_c(s) = \frac{40(s+1.5)}{(s+5.5)}$$

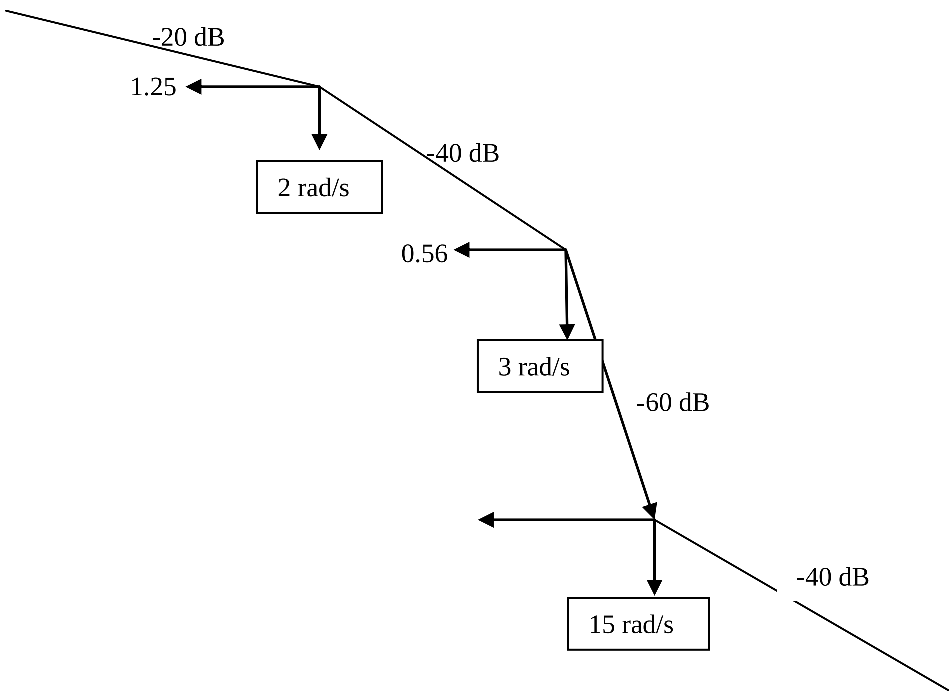
If the value of K is undetermined. -2 pts

Problem 2 (10 pts)

The open-loop transfer function of a linear time-invariant system is given by

$$A(s)=\frac{(s+15)}{s(s+2)(s+3)}=\frac{\frac{15}{6}\left(1+\frac{s}{15}\right)}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{3}\right)}$$

a. Plot the Bode diagram of the system (2 pts)



b. Determine the Gain Margin in dB. (4 pts)

At ω=3 rad/s, the angle of 180 degrees. At this frequency,

$$|A(j\omega)|=A_m==0.56$$

Therefore, $G_m = \dfrac{1}{A_m} = 1.786 = 5.036\,\text{dB}$

c. Determine the phase margin in degrees. (4 pts)

First, let us determine the cut-off frequency

$$\frac{|A(j\omega_c)|}{|A(j2)|} = \left(\frac{\omega_c}{2}\right)^{-2} \Rightarrow \frac{1}{1.25} = \frac{4}{\omega_c^2} \Rightarrow \omega_c = 2.236\,\text{rad/s}$$

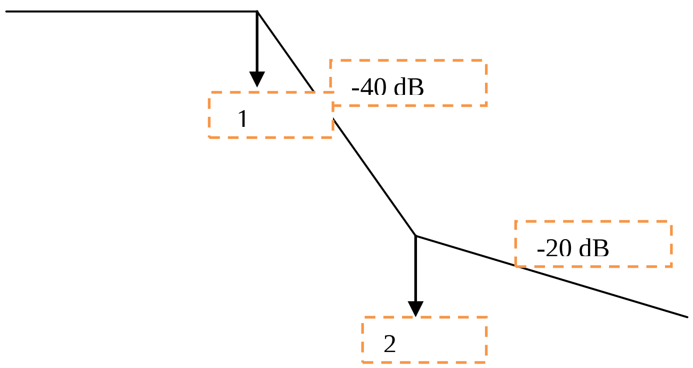
$$\beta(\omega_c) = -90^0 + \tan^{-1}(2.236/15) - \tan^{-1}(2.235/2) - \tan^{-1}(2.236/3) = -166.4^0$$

$$\theta_m = 180^0 + \beta(\omega_c) = 13.6^0$$

<u>Problem 3 (6 pts)</u>

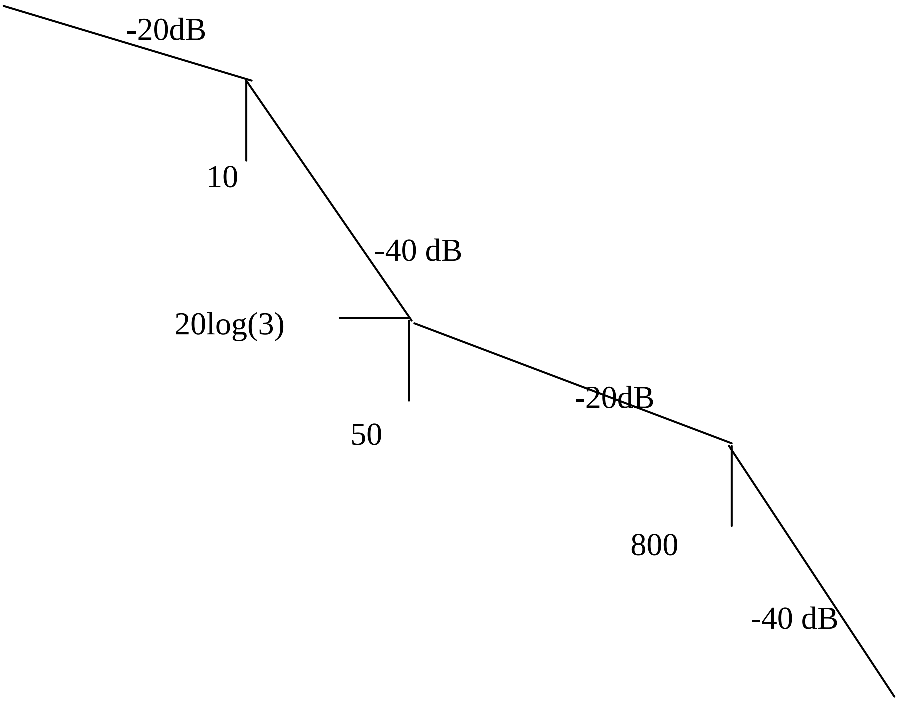
Plot the Bode diagram of a unity feedback control system whose open-loop transfer function is given by:

$$G_p(s) = \frac{2(s+2)}{(s^2-1)}$$



-2 for every incorrect slope
-2 for every incorrect Frequency

Problem 4 (8 pts)



The asymptotic gain versus frequency of the open minimum-phase transfer function is shown for a unity feedback control system.

a. Obtain the open-loop transfer function. (2 pts)

$$A(s) = \frac{750(1+s/50)}{s(1+s/10)(1+s/800)} = \frac{120{,}000(s+50)}{s(s+10)(s+800)}$$

-1 if the gain is missing

b. Determine the cut-off frequency. (2 pts)

$$\frac{|A(j\omega_c|)}{|A(j50)|} = \left(\frac{\omega_c}{50}\right)^{-1} \Rightarrow \frac{1}{3} = \frac{50}{\omega_c} \Rightarrow \omega_c = 150 \text{ rad/s}$$

c. Determine the phase margin. (2 pts)

$$\beta(\omega_c) = -90 + \tan^{-1}(150/50) - \tan^{-1}(50/10) - \tan^{-1}(50/800)$$

$$= \text{-}115.24^0$$

$$\theta_m = 180 + \beta(\omega_c) = 180 - 115.24^0 = 64.75^0$$

d. Determine the gain margin

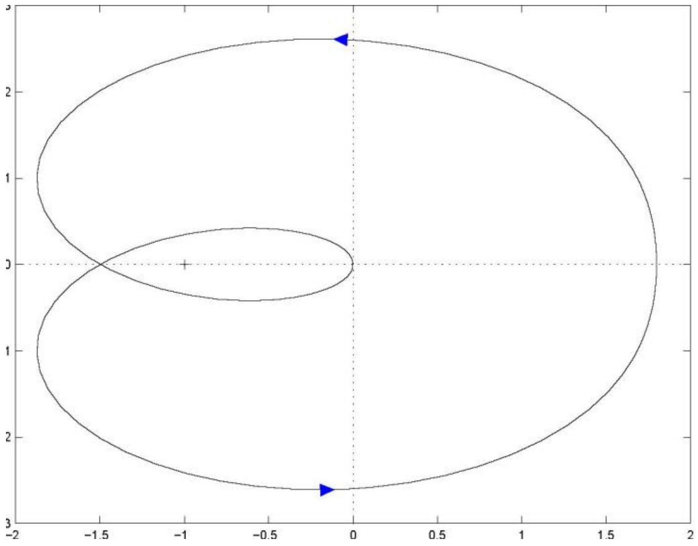
$$\prec A(j\omega) = 180^0 \text{ for } \omega > 800 \text{ rad/s} \Rightarrow \omega = \infty$$

For this value,

$$|A(j\omega| = A_m = 0 \Rightarrow G_m = \infty$$

Problem 5 (6 pts)

Consider the Nyquist plot shown below



a. What is the gain margin (in dB) for system? (3 pts)

$$G_m = \frac{1}{1.5} = 0.66$$

$$in\ \text{decibels; } \mathrm{G_m} = 20\log_{10}(0.66) = -3.521$$

If no decibels computation is done - 1

b. If the open-loop transfer function is:

$$G(s) = \frac{3(s+3)}{s^2 - 2s + 5}$$

Is the system stable? Justify your answer. (3 pts)

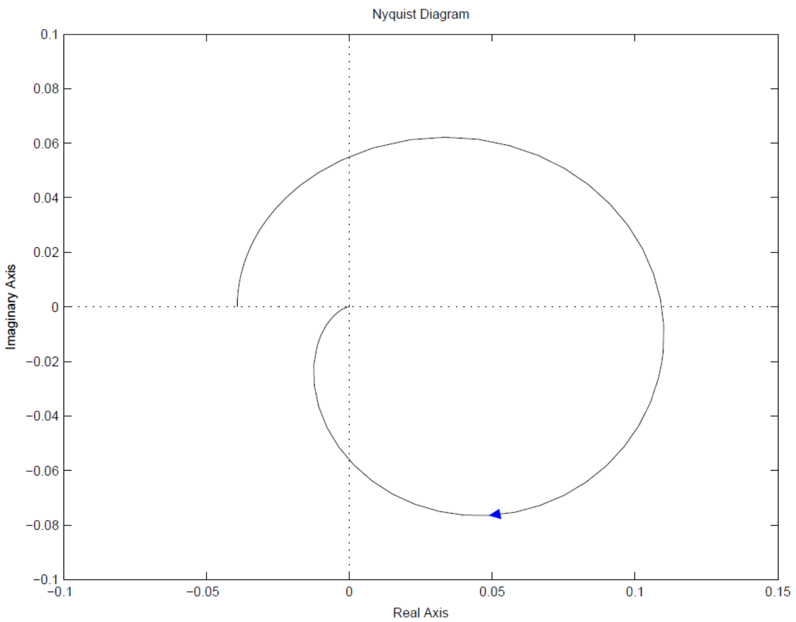
P=2 (1 pt)
N=-2 (1 pt)
Z=N+P = 0, system is stable

Problem 6 (8 pts)
Part I (4 pts)

The open-loop transfer function of a unity feedback control system is given by

$$G_p(s) = \frac{s-2}{(s+3)(s^2+2s+17)}$$

The Nyquist diagram for positive frequencies of the above system is shown below



a. On the graph draw the Nyquist diagram for all frequencies (2 pts)
b. Determine the system gain margin (2 pts)

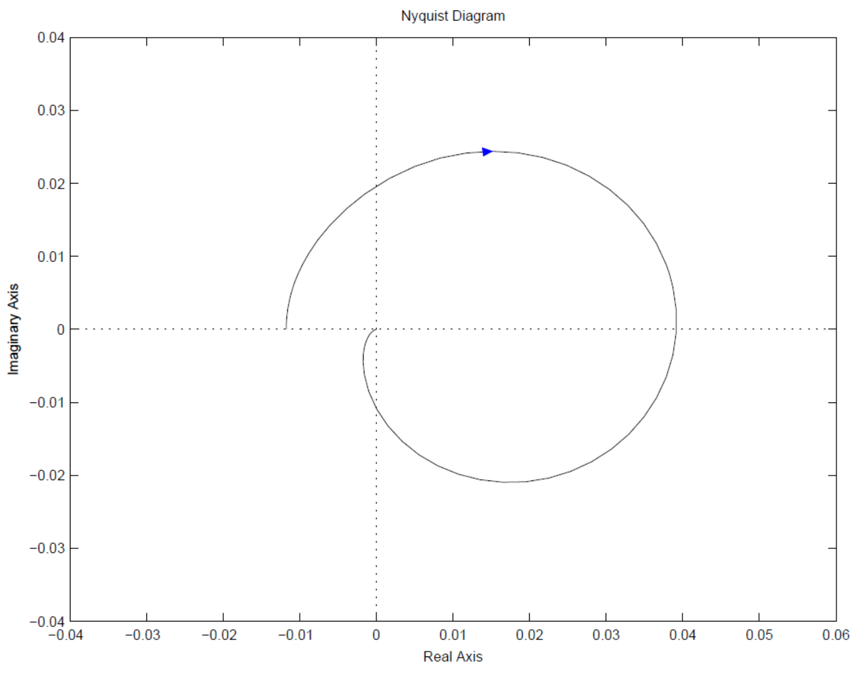
$$G_m = \frac{1}{A_m} = \frac{1}{0.044} \approx 22.73$$ **2 pts**

Part II (4 pts)

A Phase-Lead compensator of transfer function

$$G_c(s) = \frac{s+6}{s+20}$$

is added in series with the original plant. The Nyquist Diagram, for positive frequencies of the new open loop transfer function is shown below



Comments on the results

Better relative stability